THE BEHAVIOR OF THE SOIL UNDER FOUNDATION OF OFFSHORE GRAVITY STRUCTURES SUBJECTED TO DIFFERENT COMBINATION OF LOADS

Abstract. This study aims at analyzing the behavioral rules of the soil under the bottom of the offshore gravity structures when it had been subjected to the combination of load types; in which the problem of determining the deformation of the ground in semi-infinite space is determined by the view of deformed solid mechanics and elastic theory. The ground plane displacement of the soil is simulated by the computer when the structural system is subjected to concentrated loads and the distribution load is consist of the eccentricity due to wave load and other horizontal loads.

Keywords: Offshore gravity structures, Semi-infinite space, Solid mechanics.

1. The foundation of the offshore gravity construction

Offshore gravity construction is a type of construction designed according to the principle of shallow foundation, lying stably on the seabed thanks to its own weight and use load. The greater the weight of the structure, the lower the center of gravity of the structure, the more stable the structure. To achieve the above purpose, people often expand the size of the foundation base of the structure.

The expansion of the foundation base to lower the center of gravity of the structure will reduce the stress at the bottom of the foundation and increase the stability of the structure when wave loads are applied. Usually the piers of the offshore gravity construction are large in the lower part and small in the upper part.

2. The bearing capacity of the foundation when subjected to a central compressive load
With the vertical load at the center, the soil under the foundation will be forced to swell (Fig. 1). When the compressed area spreads, the foundation becomes unstable. When the shear stress in the soil reaches a critical value, it leads to a state of shear instability.

From the general unstable condition of the ground:

\[ q - \frac{\pi d^2}{4} < \pi d \int_0^L sdy + w_d L \]

the bottom limit value of the foundation can be approximated by the following soil mechanics formula:

\[ q_{gh} = N_c c + \left(1/2\right)N_{\gamma_d} B \]

where \( c \) is the adhesive force; \( \gamma_d \) is the specific gravity of the soil including buoyancy due to water action; \( B \) is the dimension characteristic of the foundation on the plan; \( N_c \) và \( N_{\gamma_d} \) are coefficients that depend on the internal friction angle of the soil; For a square foundation, \( B \) is taken to be equal to the side of the square; for rectangular foundation, \( B \) is taken as long side; for a circular foundation, \( B \) is taken as the radius;

According to Thomas H. Dawson, for sandy soil, \( c = 0 \), then formula (1) is replaced by:

\[ q_{gh} = \left(1/2\right)N_{\gamma_d} B \]

For clay can get \( \varphi=0 \); \( N_c=5.1 \); \( N_{\gamma_d}=0 \) in the original state not drained; \( c=0 \) in a drained state, then formula (1) is reduced to two forms below:

- With undrained state
  \[ q_{gh} = 5.1c \] \hspace{1cm} (3.a)

- With drainage state
  \[ q_{gh} = 0.5N_{\gamma_d} B \] \hspace{1cm} (3.b)

Usually, the undrained state is characterized by offshore structures nearing completion, when there is not enough time to reduce soil pressure. It is the drainage state that provides the long-term bearing capacity of the ground that it acquires after
draining the groundwater. The ultimate bearing capacity of the ground that has been
determined by formulas (2) and (3) is usually reduced in the calculation by a factor
of safety received in the range from 2.5 to 3.0.

Fig. 1. **Expansion of the ground due to centered load on the foundation**

3. General load-bearing gravity foundation

When the load is correctly centered vertically, we consider the pressure
transmitted to the soil to be evenly distributed over the bottom of the foundation and
use the statistical data. In particular, if this pressure is less than the allowable value
of the load-carrying capacity of the foundation, then the chosen foundation size is
acceptable to use, otherwise they are not used.

However, the foundation of offshore structures is often subjected to not only
vertical loads due to the weight of the structure, but also lateral forces and bending
moments due to water and wind movement acting on the superstructure. The
horizontal force causes a shear stress in the soil under the foundation, while the
vertical force and moment cause in the soil an unevenly distributed pressure.

According to the stable condition of the designed foundation, the value of the
shear stress caused by the lateral load is usually very small compared to the load-
bearing capacity $f_{ben}$ of the ground foundation at the bottom of the foundation when
sliding and is determined by the *Culong* formula.

To check the stability and load-carrying capacity of the ground, we can take
the maximum value of the pressure in the soil as the load distributed over all area of
the foundation bottom.

To ensure the stability of the foundation, this pressure must be less than the
allowable value determined on the basis of formulas (2) and (3).
4. Method recommended by the author

In the study, the author proposes the problem of determining the deformation of the ground at the bottom of the offshore gravity structure in semi-infinite space according to the deformation solid mechanics and the elastic theory.

Draw the deformation contour of the ground under the offshore gravity foundation when subjected to concentrated loads, uniformly distributed loads and eccentric loads caused by transverse forces.

4.1. Displacement of semi-infinite space caused by concentrated load

Considering displacement at a point (Flamand problem), the stress solution (4) of the problem of concentrated force acting on semi-infinite space is:

\[
\begin{align*}
\sigma_r &= -\frac{2P}{\pi r} \cos \theta, \\
\sigma_\theta &= \tau_r \theta = 0.
\end{align*}
\]

Fig. 3. **Stress in the r direction**
Combining Cauchy's formula, Hooke's law after the transformation, we can calculate the displacement in the $r$ direction ($\sigma_r$- direction see Fig. 4) as the following formula:

$$u_r = -\frac{2P}{\pi E} \ln r \cos \theta - c_1 \sin \theta + c_2 \cos \theta + \frac{(1 - \mu)P}{\pi E} \cos \theta - \frac{(1 - \mu)P}{\pi E} \theta \sin \theta.$$  \hspace{1cm} (5)

![Fig. 4. Displacement and stress](image)

The displacement in the $\theta$ direction will be:

$$v_\theta = \frac{2\mu P}{\pi E} \sin \theta + \frac{2P}{\pi E} \sin \theta \ln r - c_1 \cos \theta - c_2 \sin \theta - \frac{(1 - \mu)P}{\pi E} \theta \cos \theta + c_3 r.$$ \hspace{1cm} (6)

To determine the constants $c_1$, we need to keep it as an absolute solid, for example using the following condition:

- When $\theta = 0$ and for any point $r$ (point on the x-axis), then the horizontal displacement $v_\theta = 0$, from $v_\theta$ above we have $0 = -c_1 + c_3 r$. So $c_1 = c_3 = 0$

- When $\theta = 0$ and $r = h$ are quite large (point is on x-axis with depth $h$ is also quite large), then vertical displacement $u_r = 0$, from $u_r$ above, we have:

$$0 = -\frac{2P}{\pi E} \ln h + c_2 + \frac{(1 - \mu)P}{\pi E} \Rightarrow c_2 = \frac{2P}{\pi E} \ln h - \frac{(1 - \mu)P}{\pi E}.$$

Substituting $c_1$, $c_2$, $c_3$ into the above formula, we have:

Displacement $u_r$ in $r$ direction in case of concentrated load $P$ is:

$$u_r = -\frac{2P}{\pi E} \ln r \cos \theta + \left[ \frac{2P}{\pi E} \ln h - \frac{(1 - \mu)P}{\pi E} \right] \cos \theta + \frac{(1 - \mu)P}{\pi E} \cos \theta - \frac{(1 - \mu)P}{\pi E} \theta \sin \theta.$$

Or
The displacement in the $\theta$-direction, denoted by $v_\theta$, will be:

$$v_\theta = \frac{2\mu P}{\pi E} \sin \theta + \frac{2P}{\pi E} \sin \theta \ln r - \left[ \frac{2P}{\pi E} \ln \frac{h}{r} \frac{(1-\mu)P}{\pi E} \theta \sin \theta - \frac{(1-\mu)P}{\pi E} \theta \cos \theta \right]$$

Or

$$v_\theta = \frac{2P}{\pi E} \sin \theta \ln \frac{h}{r} + \frac{(1+\mu)P}{\pi E} \sin \theta - \frac{(1-\mu)P}{\pi E} \theta \cos \theta.$$  

(8)

From formula (8), calculate the vertical displacement at points lying on the horizontal boundary $x = 0$ as follows:

$$\theta = \frac{\pi}{2} \rightarrow v_\theta = \frac{2P}{\pi E} \ln \frac{h}{r} + \frac{(1+\mu)P}{\pi E}$$

$$\theta = -\frac{\pi}{2} \rightarrow v_\theta = \frac{2P}{\pi E} \ln \frac{h}{r} - \frac{(1+\mu)P}{\pi E}$$  

(9)

So the right half $y > 0$ axis subsidence is positive, so the first term that changes with $r$ always goes down, and the second term always goes up and is equal to constant. So the right branch with $r > 0$ ($y > 0$) we draw the function:

$$\theta = \frac{\pi}{2}, r > 0 \rightarrow v_\theta = \frac{2P}{\pi E} \ln \frac{h}{r} + \frac{(1+\mu)P}{\pi E}.$$  

In which, the first term with a negative sign is drawn downwards, and the second term with a positive sign is drawn back up.

Similar to the left branch $r < 0$ ($y < 0$) we draw the following function:

$$\theta = -\frac{\pi}{2}, r < 0 \rightarrow v_\theta = -\frac{2P}{\pi E} \ln \frac{h}{r} + \frac{(1+\mu)P}{\pi E}.$$  

Finally, just draw the right branch and then symmetric about the vertical axis to get the left branch.

According to Saint Venant’s principle, formula (9) of course does not recognize $r$ very close to placing $P$.

4.2. Calculate displacement in $r$ direction with uniformly distributed load $q$

When calculating the foundation of the offshore gravity structure, the uniformly distributed load $q$ can be caused by a concentrated force $P$ at the center.
This is the Flamand problem, the stress solution has been solved in the theory of elasticity. However, the problem of calculating displacement when semi-infinite space under distributed load has not been mentioned before.

From (Fig. 5) we take the differential of the external force \( q \) distribution, we have:

\[
dP = q(y)dy = q(y)(rd\theta/cos\theta)
\]

with \( dP \) will differentiate \( dr \), replace \( dr \) in (7) and integrate with variable \( \theta \), we have:

\[
u_r = \frac{2q}{\pi E} r \ln \frac{r}{H} \int_0^{\theta_0} d\theta - \frac{(1-\mu)qr}{\pi E} \int_0^{\theta_0} \theta \sin \theta d\theta = \frac{2q}{\pi E} r \ln \frac{r}{H} (\theta_2 - \theta_1) - \frac{(1-\mu)qr}{\pi E} \int_0^{\theta_0} \theta \times t g \theta \times d\theta
\]

(10)

Fig. 5. Displacement in semi-infinite space due to distributed force \( q \)

Integrating at the 2nd term in (10) is calculated by analyzing the function under the integral sign into the series, then integrating.

The result contains the Becnuli coefficients \( (Bn) \)

\[
\int x t g (ax)dx = \frac{ax^3}{3} + \frac{a^3x^5}{15} + \frac{2a^5x^7}{105} + \frac{17a^7x^9}{2835} + \ldots + \frac{2^{2n}(2^{2n} - 1)B_n a^{2n-1}x^{2n+1}}{(2n+1)!}
\]

(11)

The final formula for calculation in a computer program is:

\[
u_r = \frac{2q}{\pi E} r \ln \frac{r}{h} (\theta_2 - \theta_1) - \frac{(1-\mu)qr}{\pi E} \left[ \frac{(\theta_2^3 - \theta_1^3)}{3} + \frac{(\theta_2^5 - \theta_1^5)}{151} + \frac{2(\theta_2^7 - \theta_1^7)}{105} \right]
\]

(12)

How to calculate displacement of points on a circle with diameter \( d=2R \) (Fig. 6)
The parameters in (Figure 6) have the following relationship:

\[ r = d \cos \theta, \quad h = r \cos \theta, \quad MN = r \sin \theta, \quad \text{Tang} \theta = MN/h = \sin \theta \cos \theta. \]

– Case 1: \( z = a - MN \) (I)
– Case 2: \( z = MN - a \) (II)

Therefore, changing the sign for angle \( \theta \) for case 1 also means for case 1 so that \( z \) in case 1 also changes sign to (I) to (II). From there, we only need to use the same expression \( z \) (II) is enough.

The equation of the circle in the selected axes is:

\[
x = \frac{d}{2} \cos t + \frac{d}{2}, \quad y = \frac{d}{2} \sin t \Rightarrow r = \sqrt{x^2 + y^2}
\]

where \( t = 0^\circ-180^\circ \) rotates counterclockwise from the negative direction of the \( x \)-axis to the \( y \)-axis.

4.3. Calculate displacement in the \( r \) direction for the general case of uniformly distributed load \( q \) and moment \( M \)

When calculating the foundation of the offshore gravity structure, the moment \( M \) is caused by transverse forces.

We decompose the moment into a torque \( M = Pe \) with \( e = b \) (Fig. 7)
5. Calculation results by computer program

Use the above formula to calculate the displacement $u_r$ at $M$ belonging to circles of diameter $d$ tangent to the origin $O$, caused by $q$ and $P$ simultaneously, and then deduce the displacement at the point $N$ located to the left of the $x$ axis.

When programming, we still take the horizontal axis of the screen as the $x$-axis and the $y$-axis as the vertical.

The whole calculation process has been programmed and plotted into a contour map $u_r$ of displacement according to the above algorithm with different cases (Fig. 8).

Fig. 7. Calculating displacements in semi-infinite space caused by $q$ and $M$

Fig. 8. Displacement due to load $P$ (vertical at center)
6. Conclusions

To analyze the behavior of the ground at the bottom of the offshore gravity structure according to the Flamand problem model in Elastic Theory, the total vertical loads must be reduced to uniformly distributed loads, the total loads The horizontal shall be reduced to a torque consisting of two opposing vertical forces whose point is located at the outer edge of the perimeter of the structure's foundation.

Determining the displacement of the ground at the bottom of the offshore gravity structure according to the Flamand problem model in Elastic Theory and drawing the contour region of the displacement is a reliable new calculation method.

The elliptic integral \( \int x \tan(ax) \, dx \) is taken iff analyzing the function under the integral sign into a series, then integrating through functions containing Bernoulli's coefficient \( B_n \).

Calculating the behavior of the ground under the offshore gravity structure is very important for the design and construction of this type of structure. It is applied to the construction of oil and gas rigs, lighthouses, offshore docks, etc.

References: