MODELING OF DYNAMICS OF ELECTRODYNAMIC SPEAKER

Abstract. This article discusses an electromechanical system in the form of an electrodynamic loudspeaker. To describe the dynamics of a loudspeaker, a finite-dimensional model with a given number of a finite number of independent mechanical and electrical parameters is used. Using the electromechanical analogy force - voltage and using the Lagrange-Maxwell equations, a closed system of two nonlinear differential equations of the second order of motion of the system is compiled. Using modern advances in computer technology, a number of numerical experiments were performed in the Mathcad system with varying values of the input parameters of the system, such as the mass of the moving coil, inductance, ohmic resistance, and graphs of the time variation of the coil movement and current strength were plotted. Phase portraits were also built. The movement of the system has the character of damped oscillations, and the phase portraits have a stable focus. The influence of the varied parameters on the period of damped oscillations and the transition of the system to the state of equilibrium is shown. The study was carried out for a nonlinear model without the use of asymptotic methods, which made it possible to exclude the methodological error of the solution. The model can exhibit complex dynamics. Having a mathematical model and a calculation program, it is possible to carry out further studies of the system under consideration, identifying the positions of stable and unstable equilibrium, modes of self-oscillations, defining areas of periodic and chaotic modes of different nature. The results obtained can be used in the development of technical devices capable of demonstrating complex behavior. Methodologically, the proposed material is interesting for undergraduate and graduate students specializing in electromechanics, in terms of teaching the principles of constructing and analyzing electromechanical systems.

Keywords: nonlinear dynamics, oscillations, electro-mechanical systems, Lagrange-Maxwell equations, electrodynamic loudspeaker, mathematical model, numerical experiment, phase portrait.

Electromechanical systems are widely used in many fields of technology. For the rational design and further analysis of the properties of such systems, modern
engineering practice requires the creation of correct mathematical models, which must contain differential equations of mechanical motion, as well as the equations of electromagnetic processes. The apparatus of analytical mechanics is very convenient for compiling the equations of electromechanical systems, in which the electrical and mechanical quantities that characterize the system appear as formally equal and the equations of motion are derived using Lagrangian formalism.

This paper considers the electromechanical system of the electrodynamic speaker, to describe the dynamics of which you can use finite-dimensional models, ie models that require a finite number of independent mechanical and electrical parameters. The main attention is paid to the stage of compiling a closed system of differential equations of motion of the electromechanical system. The electromechanical analogy of force - voltage is used.

The basis of the study is mathematical modeling, which, using modern advances in computer technology, makes it possible to replace the study of complex electromechanical energy converters relatively simple for practical implementation of the model.

Consideration of this issue has been and is given quite a lot of attention [1-7]. The simulation itself was carried out using various application packages (PPPs), such as Simulink (MATLAB environment), Mathcad, MARS, Maple, Wolfram mathematica, and others. There is a wide variety of types of electromechanical systems and approaches to modeling their work. The use of one of the approaches for modeling the operation of one of the types of electromechanical systems, namely - electrodynamic loudspeaker [8-9] and the proposed work is devoted.

The aim of the work is to create and study with the help of mathematical models of the electromechanical system of an electrodynamic speaker in the Mathcad environment.

In fig. 1 schematically shows a magnetizing speaker consisting of an iron core 1 on which a magnetizing coil 2 is wound. A movable coil 3 (with a small gap) is put on the protruding end of the core, the outer edge of which is mounted on fixed brackets 5. If the applied electromotive force (EMF) $E$ changes with sound frequency, the moving coil under the action of the resulting ponderomotor forces
begins to oscillate, shifting something along the core and driving the diffuser.

Fig. 1. Electrodynamic loudspeaker: a) - calculation scheme;
     b) - general view

Let’s make the equation of motion of this system. Let $L$, $R$, $t$, $x$ - self-induction coefficient, resistance, mass and displacement coordinate of the moving coil, $L_1$, $R_1$ - the corresponding parameters of the stationary coil, $M(x)$ - coefficient of mutual induction of coils, $k$ - diffuser stiffness, $h$ is the coefficient of viscous friction.

Introducing the generalized coordinates $x$, $q = \int idt$ and $q_1 = \int i_1dt$, we write the Lagrange function of the system

$$L = \frac{1}{2} [L_1 i_1^2 + 2M(x) i i_1 + Li^2 + m \dot{x}^2 - k (x - x_0)^2].$$

Virtual work of third-party and dissipative forces

$$\delta A = (E - Ri) \delta q + (E_1 - R_1 i_1) \delta q_1 - h \dot{x} \delta x.$$

Composing the Lagrange equation of the 2nd kind, we obtain the following equations of motion:

$$\begin{aligned}
L_1 \frac{d}{dt} \frac{d}{dt} i + M \frac{d}{dt} \frac{d}{dt} i + R_1 i_1 &= E_i, \\
L \frac{d}{dt} \frac{d}{dt} i + M \frac{d}{dt} \frac{d}{dt} i_1 + R i &= E(t), \\
m \frac{d^2 x}{dt^2} + h \frac{dx}{dt} + k(x - x_0) - \frac{dM}{dx} i_i i &= 0.
\end{aligned}$$

(1)

In the first equation (1), which we write in the form
the second term in parentheses can be neglected compared to the first term, as usually \( Mi << L_i i_1 \). Assuming \( E_i = \text{const} \), we find from this equation
\[
i_1 = \frac{E_1}{R_1} = \text{const}.
\]

Consider an autonomous system, putting \( E(t) = 0 \). For small deviations \( \xi = x - x_0, \eta = i \) from the equilibrium position \( (i = 0, x = x_0) \) we can take \( M(x) = a(x - x_0) \) and the equation of motion (1) will be written as
\[
\begin{align*}
m \ddot{\xi} + h \dot{\xi} + k \xi &= a \eta, \\
L \ddot{\eta} + R \eta + a \xi &= 0.
\end{align*}
\]

It follows that the equilibrium position is always stable. If in the second equation the member \( L \dot{\eta} \) is neglected in comparison with other members, the system will be described by the equation we of the second order. Critical attenuation in the system is carried out at the following ratio:
\[
R(\sqrt{km - h}) = a^2.
\]

To model the system, we set specific values of input parameters and initial conditions: \( h = 0.16 \text{ c}^{-1}, k = 80 \text{ N/m}, a = 3 \text{ (Ohm/s)}^{1/2}, \eta_0 = 0.01 \text{ A}, \xi_0 = 0.01 \text{ m}, \xi'_0 = 0.001 \text{ m/s}. \)

The results of calculations in the form of graphs are given below.

Fig. 2. Graph of the dependence of the coordinate \( \xi(t) \) on the mass \( m \)

at \( L = 4 \text{ mH} \) and \( R = 6 \text{ Ohm} \): \( \xi_1(t) - m = 0.04 \text{ kg}; \xi_2(t) - m = 0.08 \text{ kg}, \xi_3(t) - m = 0.12 \text{ kg}; \xi_4(t) - m = 0.16 \text{ kg} \)
From the given fig. 2 shows that the mass of the moving coil significantly affects its movement. The smaller the mass, the faster the system comes to rest (in our case, with the selected parameters - about a quarter of a second).

![Graph of the dependence of the coordinate ξ(t) on the inductance L at m=0.08 kg and R=6 Ohm: ξ₅(t) - L=0.5 mH; ξ₆(t) - L=1 mH; ξ₇(t) - L=2 mH; ξ₈(t) - L=4 mH.](image)

Fig. 3. Graph of the dependence of the coordinate ξ(t) on the inductance L at m=0.08 kg and R=6 Ohm: ξ₅(t) - L=0.5 mH; ξ₆(t) - L=1 mH; ξ₇(t) - L=2 mH; ξ₈(t) - L=4 mH

Fig. 3 indicates that the coordinates ξ(t) depend little on the inductance L.

Resistance R significantly affects the time of bringing the system to rest (Fig. 4). At resistance R=4 Ohms and the set parameters the system comes to a state of rest for 0,25 seconds.

The speed of the moving coil v(t) = ξ'(t).

Figures 5-7 show the phase portraits of the system when varying the input parameters.

![Graph of the dependence of the coordinate ξ(t) on the resistance R at m=0.08 kg and L=4 mH: ξ₉(t) - R=4 Ohm; ξ₁₀(t) - R=8 Ohm; ξ₁₁(t) - R=12 Ohm; ξ₁₂(t) - R=16 Ohms.](image)

Fig. 4. Graph of the dependence of the coordinate ξ(t) on the resistance R at m=0.08 kg and L=4 mH: ξ₉(t) - R=4 Ohm; ξ₁₀(t) - R=8 Ohm; ξ₁₁(t) - R=12 Ohm; ξ₁₂(t) - R=16 Ohms
Fig. 5. Phase portrait of the system depending on the coordinate $\xi(t)$ from the mass $m$ at $L=4 \, mH$ and $R=6 \, Ohm$: $\xi_1(t) - m=0.04 \, kg$; $\xi_2(t) - m=0.08 \, kg$; $\xi_3(t) - m=0.12 \, kg$; $\xi_4(t) - m=0.16 \, kg$

Fig. 6. Phase portrait of the system depending on the coordinate $\xi(t)$ from the inductance $L$ at $m=0.08 \, kg$ and $R=6 \, Ohm$: $\xi_5(t) - L=0.5 \, mH$; $\xi_6(t) - L=1 \, mH$; $\xi_7(t) - L=2 \, mH$; $\xi_8(t) - L=4 \, mH$
Fig. 7. Phase portrait of the system depending on the coordinate $\xi(t)$ from the resistance $R$ at $m=0.08$ kg and $L=4\ mH$: $\xi_9(t) - R=4\ Ohm;\ \xi_{10}(t) - R=8\ Ohm;\ \xi_{11}(t) - R=12\ Ohm;\ \xi_{12}(t) - R=16\ Ohms$

In all cases (Figures 5-7) phase trajectories have a stable focus.

The nature of the change in current (the second generalized coordinate) over time depending on the varied parameters can be seen from Figures 8-10.

Fig. 8. Graph of the dependence of the coordinate $\eta(t)$ on the mass $m$ at $L=4\ mH$ and $R=6\ Ohm$: $\xi_1(t) - m=0.04\ kg;\ \xi_2(t) - m=0.08\ kg;\ \xi_3(t) - m=0.12\ kg;\ \xi_4(t) - m=0.16\ kg$
Fig. 9. **Graph of the dependence of the coordinate $\eta(t)$ on the inductance $L$ at $m=0.08$ kg and $R=6$ Ohm:**
- $\zeta_6(t)$ - $L=0.5$ mH;
- $\zeta_7(t)$ - $L=1$ mH;
- $\zeta_8(t)$ - $L=2$ mH;
- $\zeta_9(t)$ - $L=4$ mH

Fig. 10. **Graph of the dependence of the coordinate $\eta(t)$ on the resistance $R$ at $m=0.08$ kg and $L=4$ mH:**
- $\zeta_{10}(t)$ - $R=4$ Ohm;
- $\zeta_{11}(t)$ - $R=8$ Ohm;
- $\zeta_{12}(t)$ - $R=12$ Ohm;
- $\zeta_{13}(t)$ - $R=16$ Ohms

Figures 8-10 show that according to the second generalized coordinate, the system reaches a state of rest in a short period of time.

**CONCLUSIONS**

Modeling the dynamics of electromechanical systems based on Lagrange-Maxwell equations and using modern PPP and computer technology can reduce material costs for the development of electromechanical devices and by varying the parameters to choose their rational combination. If you decide on the criterion of optimality, then having a mathematical model and program, you can perform optimization of parameters. The analysis of nonlinear dynamics of the loudspeaker is carried out. The study was performed for a nonlinear model without the use of
asymptotic methods, which eliminated the methodological error of the solution. Graphs of displacement and velocity parameters, as well as phase portraits are constructed.

References: