SOLUTION OF BOUNDARY-VALUE PROBLEMS FOR THE SYSTEMS OF PSEUDO-DIFFERENTIAL EQUATIONS

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The use of the results of the implementation of computational mathematical models for the optimization and control of technical and biotechnological processes has attracted the attention of many native and foreign scientists. We shall state the importance of the results obtained in the publications [1, 2] where the criteria for improving the quality of laser segmentation of the embryo and the assessing the reliability of the logistics system of local cargo transportation were developed.

In the report, the boundary value problem for a system of multidimensional, non-stationary pseudo-differential equations in a poly-layer was solved. The uniqueness of the authors' research is in taking into account the inhomogeneous structure of the objects studied and the technical parameters of the emitters. This will improve the quality of the simulated processes by reducing the costs of the objects under study and increasing the reliability of software and hardware for the implementation of applied optimization mathematical models.

We shall consider homogeneous and inhomogeneous boundary value problems:

\[
\begin{align*}
\frac{\partial u(x,t)}{\partial t} &= A_1 \left( \frac{\partial}{i \partial x} \right) u(x,t) \quad \text{at} \quad t \in [0; t_1], \quad x \in R^m; \\
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
\frac{\partial u(x,t)}{\partial t} &= A_n \left( \frac{\partial}{i \partial x} \right) u(x,t) \quad \text{at} \quad t \in [t_{N-1}; T], \quad x \in R^m.
\end{align*}
\]

(1)

\[
B_0 \left( \frac{\partial}{i \partial x} \right) u(x,0) + B_1 \left( \frac{\partial}{i \partial x} \right) u(x,t_1) + \cdots + B_n \left( \frac{\partial}{i \partial x} \right) u(x,T) = \varphi(x)
\]

(2)

and
where

\[ \frac{\partial u(x,t)}{\partial t} = A_i \left( \frac{\partial}{i \partial x} \right) u(x,t) + f(x,t) \quad \text{at} \quad t \in [0;t_1], \ x \in \mathbb{R}^n; \]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \]

\[ \frac{\partial u(x,t)}{\partial t} = A_n \left( \frac{\partial}{i \partial x} \right) u(x,t) + f(x,t) \quad \text{at} \quad t \in [t_{N-1};T], \ x \in \mathbb{R}^n. \]  

\[ B_0 \left( \frac{\partial}{i \partial x} \right) u(x,0) + B_1 \left( \frac{\partial}{i \partial x} \right) u(x,t_1) + \cdots + B_n \left( \frac{\partial}{i \partial x} \right) u(x,T) = 0. \]  

\[ A_k \left( \frac{\partial}{i \partial x} \right)_i B_k \left( \frac{\partial}{i \partial x} \right) \]

are pseudo-differential operators with symbols in the open space of indefinitely differentiated functions of power development;

\[ u(x,t) \]

are solutions to boundary value problems;

Let us act by the re-implementations of Fourier’s (at spacious wines) on equations with homogeneous and inhomogeneous boundary value problems (1)–(2), (3)–(4). Received the following:

\[ \frac{\partial u(s,t)}{\partial t} = A_i(s) u(s,t) \quad \text{at} \quad t \in [0;t_1], \ s \in \mathbb{R}^n; \]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \]

\[ \frac{\partial u(s,t)}{\partial t} = A_n(s) u(s,t) \quad \text{at} \quad t \in [t_{N-1};T], \ s \in \mathbb{R}^n. \]  

\[ B_0(s) u(s,0) + B_1(s) u(s,t_1) + \cdots + B_n(s) u(s,T) = \varphi(s) \]

and

\[ \frac{\partial u(s,t)}{\partial t} = A_i(s) u(s,t) + \tilde{f}(s,t) \quad \text{at} \quad t \in [0;t_1], \ s \in \mathbb{R}^n; \]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \]

\[ \frac{\partial u(s,t)}{\partial t} = A_n(s) u(s,t) + \tilde{f}(s,t) \quad \text{at} \quad t \in [t_{N-1};T], \ s \in \mathbb{R}^n. \]  

providing that:

\[ B_0(s) u(s,0) + B_1(s) u(s,t_1) + \cdots + B_n(s) u(s,T) = 0, \]

where \( u(s,t) \) is the transformation of Fourier’s function \( u(x,t) \);

\[ \tilde{\varphi}(s) \]

is the transformation of Fourier’s function \( \varphi(x) \);

\[ A_k(s), \ B_k(s) \]

are symbols of the appropriate pseudo-differential operators;

\[ \tilde{f}(s,t) \]

is the transformation of Fourier’s function \( f(x,t) \).

There by, the solution of the tasks (5)–(6) will look like:
The authors explored the problems of development and numerical implementation of computational mathematical models of multilayer systems under the influence of physical fields sources. The use of the proposed algorithm for the realization of boundary value problems of the systems of differential and pseudo-differential equations in a poly-layer will allow to increase the accuracy of solving applied problems of calculating and optimizing the parameters of technical and biotechnological systems by taking into account the structural features of objects of the simulated systems and technical parameters of emitters.

References:
